Final Exam – Review 2 – Problems

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1 The definition of the integral

Problem 1

Use the definition of the integral to evaluate $\int_1^2 x^2 dx$

> **Note:** You may use $\sum_{i=1}^n i = \frac{n(n+1)}{2}$ and $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$

Problem 2

Use the midpoint rule with $n = 5$ to approximate $\int_0^{10} x^2 dx$

Problem 3

Evaluate the following limit:

$$\lim_{n\to\infty} n^2 \left(\frac{1}{(n+1)^3} + \frac{1}{(n+2)^3} + \cdots + \frac{1}{(n+n)^3} \right)$$

2 Calculating integrals

Problem 4

Evaluate the following integrals:

(a)

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

(b)

$$\int_{-1}^1 \tan\left(\frac{\pi}{4}x\right) dx$$

(c)

$$\int \frac{\left(\tan^{-1}(x)\right)^2}{1+x^2} dx$$

(d)

$$\int_1^2 x\sqrt[4]{x-1}dx$$

(e)

$$\int \frac{x+2}{4\,x^2+1}dx$$

(f)

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos(x)}{\sin^2(x)}dx$$

3 Differentiating integrals

Problem 5

Find the antiderivative F of $\sin\left(x^2\right)$ which satisfies $F(2) = 3$

Problem 6

Differentiate $g(x) = \int_{e^x}^{\sin(2x)} \sin^{-1}(t)dt$

Problem 7

Show that the following integral does not depend on x:

$$I(x) = \int_0^x \frac{dt}{1+t^2} + \int_0^{\frac{1}{x}} \frac{dt}{1+t^2}$$

4 Areas

Problem 8

Find the area of the region enclosed by the curves $x = 12 - y^2$ and $x = y^2 - 6$